Effect of Link-Level Feedback and Retransmissions on the Performance of Cooperative Networking

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***Abstract*—*Cooperative Networking* is a new technology which exploits the massive deployment of nodes in wireless sensor networks. *Cooperative Networking* synergistically integrates *Networking* with cluster-based *Cooperative Communications* to improve reliability and enhance network performance. In this paper, we consider the effect of link-level feedback and retransmissions on the performance of wireless sensor networks using *Cooperative Networking*, and we present scenarios where link-level retransmission offers a significant improvement in network throughput. Generally, *Cooperative Networking* with link-level retransmission provides higher throughput when the network node density is low (i.e., *sparse networks*) or in environments with adverse conditions such as high probability of transmission loss and low connectivity among the nodes.**

Keywords – Cooperative Networking, Cooperative Communications, Network Coding, Cooperative Networks, Link-Level Feedback, Retransmission, Clustering.

I. INTRODUCTION

Cooperative Networking [1] is a new technology that synergistically integrates *Cooperative Communications* and *Network Coding* and improves network performance, such as throughput and reliability, primarily by reducing the probability of packet loss.

Cooperative Communications is a well-known technique that allows single-receiver devices to obtain some of the advantages of Multiple-Input-Multiple-Output (MIMO) systems [2]. As shown in [2] and [3], MIMO systems can transmit higher bit rates than Single-Input-Single-Output (SISO) systems with the same transmission power and under the same bit-error rate channel conditions.

Network Coding [4] achieves throughput gain by using spatial path diversity and by combining independent (or partially independent) pieces of information in intermediate network nodes. In one implementation of Network Coding, random coefficients are selected by the combination-producing network nodes. The random coefficients are then transmitted in the packet header. Reference [5] shows that network coding can also be used to improve network reliability, and, in particular, for recovering from failures [6]. Another advantage of Network Coding is that it increases security, since the information transmitted through the links is a random linear combination of packets that are received via different input links and it is less likely that a single node will receive sufficient information to decode all the source information.

In [1], the authors analyzed the performance of Cooperative Networking, but without link-level feedback and retransmissions. In this paper, we extend the work done in [1] by analyzing the effect of link-level feedback (i.e., packet retransmission) on Cooperative Networking. Link-level feedback is implemented when an insufficient number of combination packets is received at the destination node, so that the destination cannot reproduce the original packets transmitted by the source. To compare the performance of Cooperative Networking with and without link-layer feedback, we rely on two metrics: the throughput and the probability of recovery of the source information at the destination.

The paper is organized as follows. In section II, we summarize the work done in [1]. The analysis of Cooperative Networking with link-level retransmission is derived in Section III. Section IV presents numerical results of the effect of link-level retransmission on Cooperative Networking. Finally, Section V concludes this work.

II. RELATED WORK

Integration of Cooperative Communications with Network Coding in wireless cluster-based networks was proposed by Haas and Chen in [1]. This novel technology, referred to as Cooperative Networking (CN), can provide significant improvement in network performance (such as throughput and probability of delivery), as compared with the performance of either of the two schemes alone.

In cluster-based Cooperative Communications (without Network Coding), clusters are formed by grouping geographically close nodes around each node on the path from the source to the destination [1,7]. Source-generated packets are transmitted to the first cluster and then from one cluster to the next cluster towards the destination node. When a packet is transmitted by a cluster, as is described below, many, if not all, the cluster's nodes cooperate in the transmission.

Cooperative Networking synergistically combines Cooperative Communication with coding of packets via Network Coding, where the latter is typically implemented

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based on linear operations over a Galois Field. Cooperative Networking incorporates the functions of route determination, creation and control of the clusters, and cluster-to-cluster transmissions [7].

We refer to the source-generated packets as "original packets," while the coded-packets are referred to as "combination packets," or simply as "combinations." Nodes in a cluster receive combination packets from nodes of the prior cluster, create new combination packets (one new combination packet per each node), and transmit the combination packets to the nodes in the next cluster. Of course, the goal is to forward as many independent combinations as possible. We refer to a combination as being "innovative," if it is linearly independent of all the other combinations already transmitted by the nodes of the same cluster. The diagram of the network architecture is shown in Fig. 1, where there are *K* clusters and n_i nodes in the i^{th} cluster. The overall objective is for the destination node to be able to correctly reproduce the original packets.

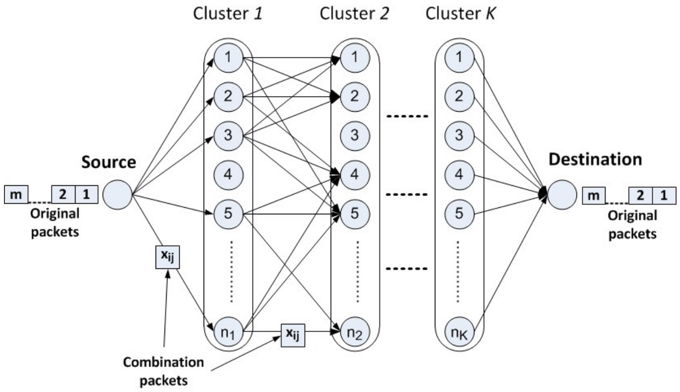


Figure 1: The Cooperative Networking model

Communications are organized in blocks of *m* packets. We label the original packets as x_k and the j^{th} combination packet that is transmitted from the cluster *i* (to the cluster *i+1*), as x_{ij}. Similarly, the j^{th} combination packet transmitted from the source to the first cluster is labeled x_{Sj}, and the j^{th} combination packet transmitted from the last cluster (the K^{th} cluster) to the destination is labeled as x_{Kj}. Thus,

$$x_{Sj} = \sum_{k=1}^{m} c_{jk} x_k \tag{1}$$

where the coefficients c_{jk} are randomly chosen from a Galois Field, $GF(2^q)$.

Since *m* is the number of original packets in a block sent by the source node, thus, *m* is the minimal number of (independent) combinations that the destination needs to receive to be able to recover all the *m* original packets. The combination packet transmitted by node *j* in the cluster *i* to the nodes in the cluster *i+1* is denoted as:

$$x_{ij} = \sum_{k=1}^{n_{i-1}} c_{ijk} x_{i-1,j} \tag{2}$$

where the coefficients c_{ij} are randomly chosen from a Galois Field, $GF(2^q)$.

In [1], the authors discussed the system's parameters and the mathematical model of Cooperative Networking. Table I lists the definitions of the system's parameters used in the discussion in [1]. In that study, the authors determined the appropriate values of the system's parameters to achieve an optimal performance of the network under the following assumptions:

- There is no link-level feedback.
- The number of original packets *m* is 10.
- All the clusters have the same number of nodes $n = n_i$
- The connectivity between node *j* in the cluster *i* and nodes in the cluster $i + 1$ is denoted as r_{ij} and, furthermore, $r = r_s = r_{ij}$.
- All the links have the same characteristics, i.e., $p = p_{(i,j)(i+1,l)}$, such that $1 \le i \le K - 1$, $1 \le j \le n_i$, and $1 \le l \le n_{i+1}$. Although this assumption may not be realistic in some network scenarios, it considerably simplifies the analysis and evaluation.

TABLE I. SYSTEM PARAMETERS [1]

Parameter	Description
n_i	Number of nodes in the cluster *i*
K	Number of clusters between the source and the destination
r_{ij}	Number of nodes in the cluster *i+1* that are connected with node *(i,j)*
r_s	Number of nodes in the cluster *1* that are connected with the source node
p_{yz}	Probability of transmission loss of a link between node *y* and node *z*
m	Number of original packets in a block (i.e., block size)

Figure 2 shows the throughput (number of correctly received packets) vs. the number of nodes per cluster (*n*) for the Cooperative Networking and the Multihop Packet networks, demonstrating the significant improvement of the former scheme. (For the Multihop Packet network case, a single path between the source and the destination is chosen and packets are forwarded along the path.) The results in this figure were calculated for the probability of transmission loss *p*=0.1.

Generally speaking, in packet networks, reliability can be improved via channel coding and retransmission schemes, both of which increase the transmission data rate. When a link fails, reliability could be achieved by rerouting the packets along an alternative route. In contrast, Cooperative Networking increases reliability by applying redundancy across the spatial domain, so that when some packets are erroneous or even completely lost, it is quite likely that the other network paths can provide sufficient information for the destination node to recover the transmitted packets. Therefore, Cooperative Networking can guard against failures of links or nodes without end-to-end retransmissions.

III. ANALYSIS OF COOPERATIVE NETWORKING WITH LINK-LEVEL RETRANSMISSION

We begin by examining, the probability of successfully decoding of a message by the destination, P_S, and the probability V_K that at least one combination packet is correctly received by a node in the cluster K. Using the assumptions made in [1], the parameter V_K is equal for all the nodes in the cluster K. The results for V_K and P_S are shown in Fig. 3 and Fig. 4.

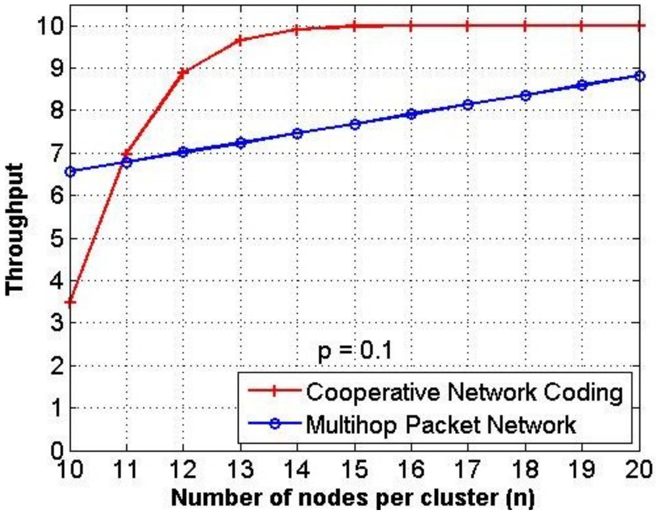


Figure 2: The throughput vs. number of nodes per cluster (n) of Cooperative Networking (with r=8) and of Multihop Packet Network

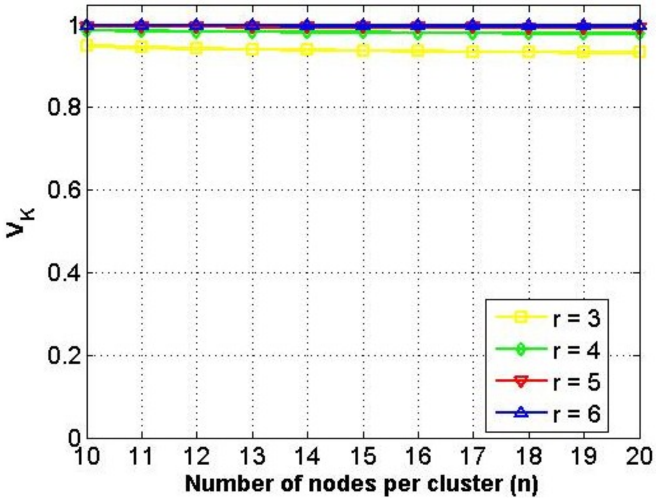


Figure 3: Probability V_K that a node in the cluster K correctly receives at least one combination packet vs. number of nodes in a cluster (n) for different values of connectivity r and for p=0.1

Fig. 3 demonstrates that, for connectivity values r greater than 3, the probability that at least one combination packet is correctly received by a node in the cluster K is close to 1, independently of the number of nodes in a cluster. However, as is shown in Fig. 4, the probability that the destination node can decode the original message is much lower than V_K for a cluster size smaller than 13 nodes. In other words, as might be expected, the performance of the links between nodes in the last cluster (the K^{th} cluster) and the destination node significantly affects the network's performance. Similar results, depicted in Figs. 5 and 6, were obtained for the probability of transmission loss p = 0.25. Thus, even when the probability of transmission loss p of a link increases to 0.25, the probability that at least one combination packet is correctly received by a node in the cluster K is still close to 1 for values of r greater than 3. However, the probability P_S that the destination node can decode the original message is significantly affected when the number of nodes in a cluster is less than 16 nodes.

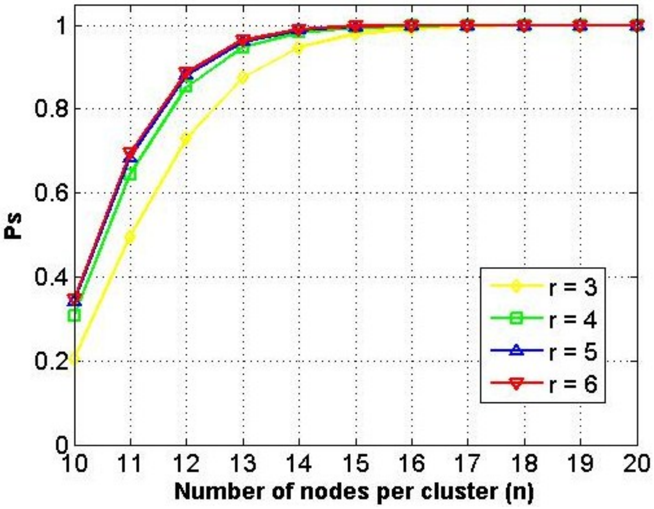


Figure 4: The probability of successful reception P_S vs. number of nodes in a cluster (n) for a number of values of connectivity (r) and for p=0.1

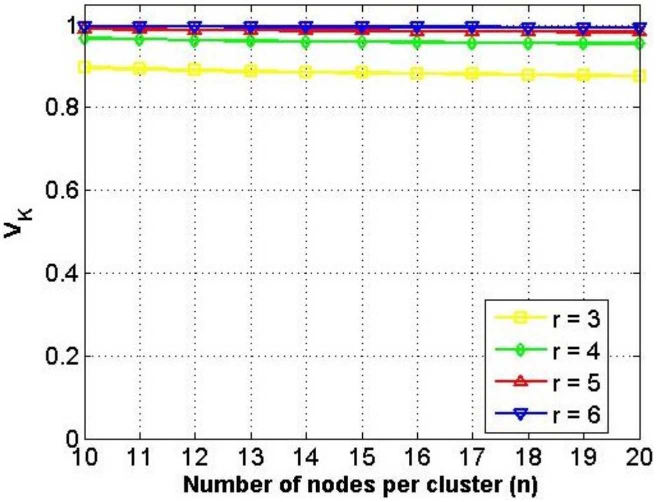


Figure 5: Probability V_K that a node in the cluster K correctly receives at least one combination packet vs. number of nodes in a cluster (n) for different values of connectivity r and for p=0.25

Additionally, the probability of successful reception P_S decreases when not all the nodes in the cluster K are connected to the destination node. For example, if three nodes of the cluster K are disconnected from the destination, we need the cluster K to be of size of at least 13 nodes to achieve the same performance as with a cluster size of 10 when all the nodes are connected to the destination node.

Since the probability V_K that at least one combination packet is correctly received by a node in the cluster K is already close to 1, it is intuitively clear that link-layer retransmissions would be of benefit only in the last hop; i.e., on the links from nodes in the cluster K to the destination node. This is an important observation, as only the feedback from the destination node to nodes in the last cluster (the K^{th} cluster) suffices, without the need for retransmission from the source node to the destination node.

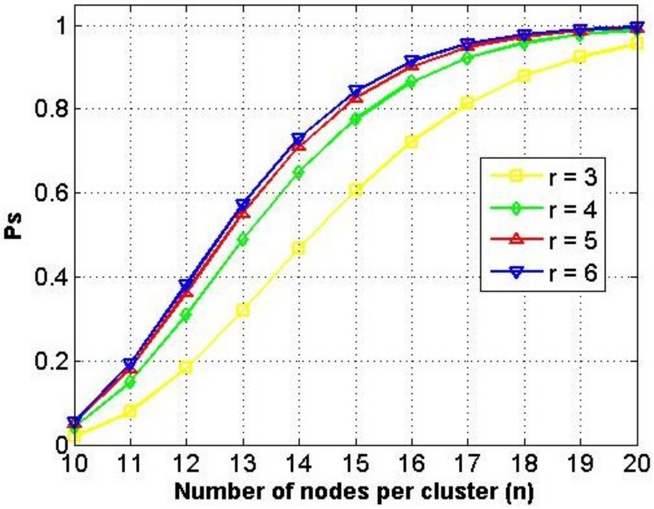


Figure 6: The probability of successful reception P_S vs. number of nodes in a cluster (n) for a number of values of connectivity (r) and for p=0.25

Extending equation (1) in [1], the probability of successful reception P_S is computed as the sum of the combinations of successful reception of the links between nodes in the cluster K and the destination node P_{di}. P_S is given by:

$$Ps = Prob(t \geq m)$$

$$Ps = \sum_{t=m}^{n_K} \left[\sum_{\vec{a} \in A} \left(\prod_{i \in a_1} \cdot \prod_{j \in a_0} P_{di} \left(1 - P_{dj}\right) \right) \right] \tag{3}$$

$$P_{di} = V_{Ki} r_{Ki} \left(1 - p_{(K,i)d}\right) \tag{4}$$

where we note that:

- A is a set of n_K binary sequences of all the 2^{n_K} possible combinations. A binary sequence can contain either 0 or 1, where "1" means that the transmission was successful and "0" otherwise. The number of 1-s in A is t and the number of 0-s is $(n_K - t)$; so there are $\binom{n_K}{t}$ such sequences. Thus,

$$\|A\| = \binom{n_K}{t} \tag{5}$$

- $\vec{a}$ is a particular sequence from the set A, a_0 is a set of all indices j of $\vec{a}$ such that $a(j) = 0$, and a_1 is a set of all indices i of $\vec{a}$ such that $a(i) = 1$. Thus $\|a_0\| + \|a_1\| = n_K$,
- P_{di} is the probability that a combination packet, transmitted from node i in the cluster K, is correctly received by the destination node,
- V_{Ki} is the probability that node i in the cluster K receives at least a combination packet from nodes of the cluster K-1,
- r_{Ki} is the connectivity between node i in the cluster K and the destination node. This parameter could be either 1 or 0,
- $p_{(K,i)d}$ is the probability of transmission loss between node i in the cluster K and the destination node.

A. *Link-Level Retransmission over the last hop*

If the destination node receives less than m correct combination packets, the destination node is unable to recover the original information. Therefore, in the scheme proposed in this paper, the destination node stores the received combination packets and requests new combination packets to be retransmitted from the K^{th} cluster, in which case, every node of the K^{th} cluster transmits a new combination packet. Successful reception occurs if the total number of correctly received packets in the original transmission and in the retransmissions equals or exceeds m. (The destination node will request such a retransmission any time that it receives at least one, but less than m combinations.) In this context, the link-level feedback means that the destination asks for retransmission from nodes in the last cluster (the K^{th} cluster). In the analysis, we account for packet loss of the retransmissions, as well as of the retransmission requests. The diagram of the link-layer retransmission scheme between nodes in the cluster K and the destination node is shown in Fig. 7.

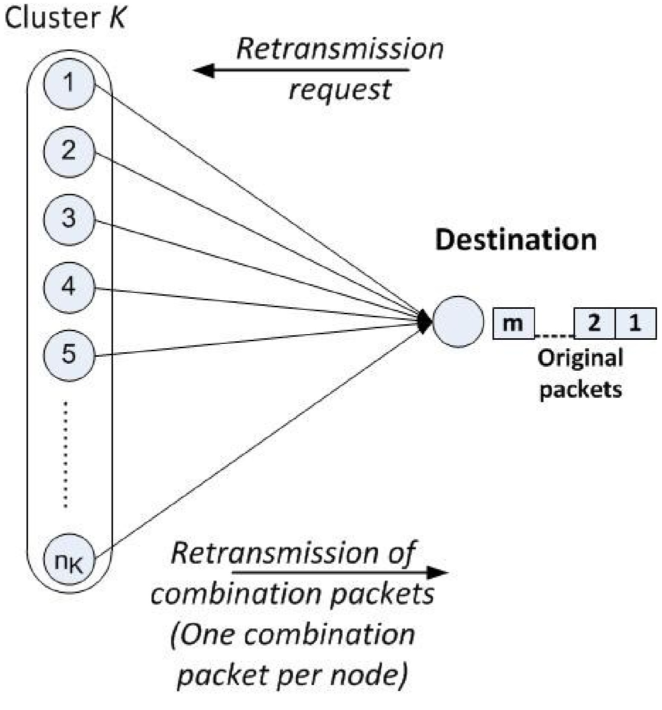


Figure 7: Link-layer retransmission model.

The probability that the destination node requests retransmission, denoted by P_r, is given by:

$$P_r = Prob(1 \leq t \leq m-1)$$

$$P_r = \sum_{t=1}^{m-1} \left[\sum_{\vec{a} \in A} \left(\prod_{i \in a_1} \cdot \prod_{j \in a_0} P_{di} \left(1 - P_{dj}\right) \right) \right] \tag{6}$$

The number of combination packets received at the destination node is represented by t and the indices

$\vec{a}, a_0, a_1, i, j$ are as defined in (3).

The probability that the node i in the cluster K correctly receives the retransmission request from the destination node (P_{Ki}) is given by:

$$P_{Ki} = (1 - BER_i)^L \tag{7}$$

where BER is the probability of a bit being in error over this link. However, since the retransmission request packet would be typically small (a few bytes) relative to a combination (data) packet, its probability of transmission loss, can be considered negligible compared to the probability of transmission loss of a combination packet (p_{ij}). Thus, the probability that a retransmitted combination packet is successfully received at the destination, denoted as P_{ret}, is given by:

$$P_{ret_i} = V_{Ki} r_{Ki} \left(1 - p_{(K,i)d}\right)$$
$$P_{ret_i} = P_{di}, \qquad 1 \leq i \leq n_K \tag{8}$$

After the second transmission, the destination node receives, in the best case, up to n_K packets in the first transmission and up to n_K packets in the retransmission.

The formula for the probability of successful reception with link-level retransmission Ps_f is given by:

$$Ps_f = \sum_{t=m}^{2n_K} \left[\sum_{\vec{a} \in A} \left(\prod_{i \in a_1} \cdot \prod_{j \in a_0} P_{di} \left(1 - P_{dj}\right) \right) \right] \tag{9}$$

under the following conditions:

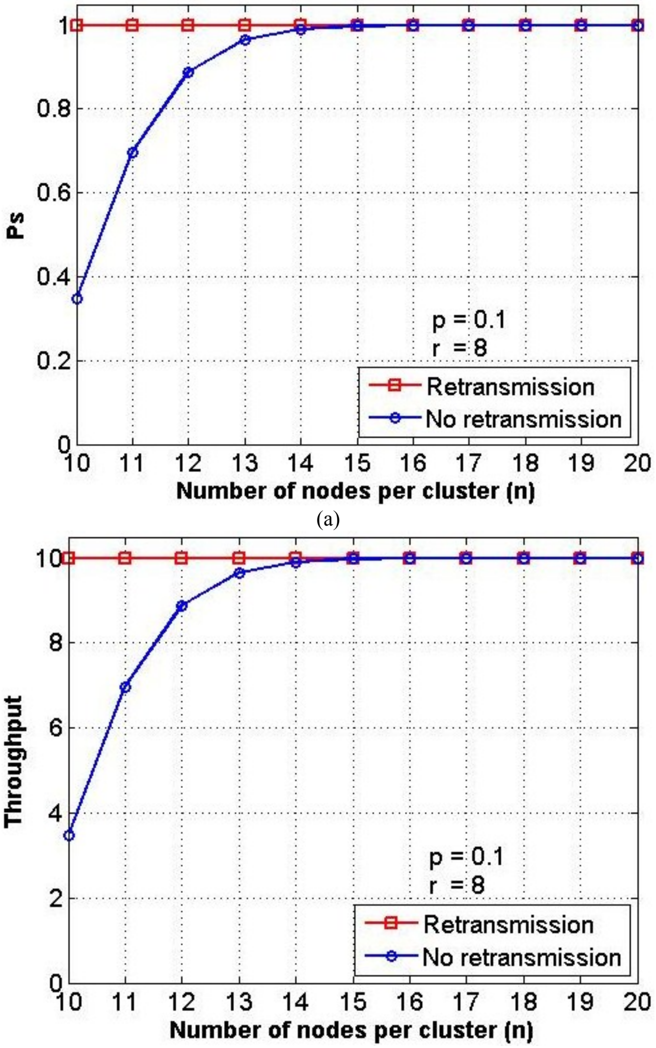
- A is a set of $2n_K$ binary sequence of all the 2^{2n_K} possible combinations. A binary sequence can contain either 0 or 1, where “1” means that the transmission was successful and “0” otherwise. The number of 1-s in A is t and the number of 0-s is $(2n_K - t)$; so there are $\binom{2n_K}{t}$ such sequences.
- $\vec{a}$ is a particular sequence from the set A, a_0 is a set of all indices j of $\vec{a}$ such that $a(j) = 0$ and a_1 is a set of all indices i of $\vec{a}$ such that $a(i) = 1$. Thus $\|a_0\| + \|a_1\| = 2n_K$,
- P_{di} is the probability that a combination packet, transmitted from node i in the cluster K, is correctly received by the destination node.

In the next section we evaluate the performance of Cooperative Networking with link-level retransmission and compare to the results obtained in [1].

IV. EVALUATION OF THE PERFORMANCE OF COOPERATIVE NETWORKING WITH LINK-LEVEL RETRANSMISSION

In our evaluations, we compared the probability of successful reception of Cooperative Networking with and without link-level retransmission.

Cooperative Networking with link-level retransmission is evaluated considering the number of original packets *m =10*, (as in [1]) and the cluster size *n* of up to 20 nodes per cluster. In particular, we assumed that the probability of error of all the links are equal.



(a)

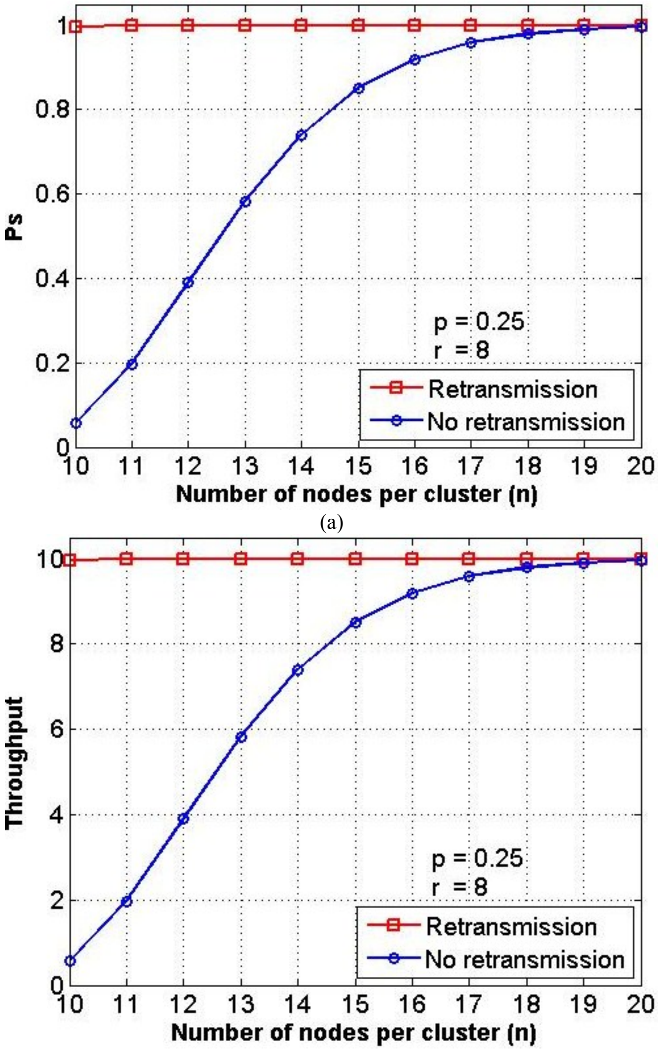
(b)

Figure 8: (a) Probability of successful reception *Ps* vs. number of nodes *n* in a cluster for different levels of retransmission, (b) Throughput vs. number of nodes *n* in a cluster for different levels of retransmission.

As we can see in Fig. 8 and 9, Cooperative Networking with link-level retransmission implemented between the last cluster (the K^{th} cluster) and the destination node has better performance than Cooperative Networking without link-level retransmission. Or course, this is intuitively clear, since the destination node can receive more combination packets with link-level retransmission.

V. CONCLUSIONS

Our study in this paper focused on analyzing the effect of link-level retransmission on the performance of Cooperative Networking. Based on the range of parameters we have investigated, Cooperative Networking with link-level retransmission offers significant performance improvement in sparse wireless sensor networks, that is when the cluster size *n* and the connectivity of the network *r* are small.



(a)

(b)

Figure 9: Comparison of link-level retransmissions with no link-level retransissions: (a) the probability of successful reception P_S vs. the number of nodes in a cluster (*n*), (b) the throughput vs. number of nodes in a cluster (*n*)

By implementing link-level retransmission in Cooperative Networking, the probability of successful reception *Ps* can be increased from 0.05 without link-level retransmissions to close to 1 with link-level retransmissions, when the number of nodes per cluster *n* is equal to the number of original packets *m* ($n = m = 10$) and the probability of transmission loss *p* is 0.25.

For cluster sizes *n* of less than 15 nodes per cluster and the connectivity of nodes *r* less than 8, link-level retransmissions offers a significant improvement in the probability of successful reception *Ps* from values in the range (0.05 to 0.35) with no link-level retransmissions, to values grater than 0.95 with link-level retransmissions.

Moreover, when not all the nodes in the cluster *K* are connected to the destination node, link-layer retransmission can help to increase the network's performance without increasing the cluster size.

Also, we observe that link-layer retransmissions on other than the last hop will not produce significant improvement in the performance of Cooperative Networking, since the probability that a node in the cluster *K* correctly receives at least one combination packet, V_K, is already close to 1. In fact, implementation of link-layer retransmissions on other than the last hop would be counter-productive, because of the unnecessary consumption of network resources and the introduction of extraneous traffic in the network.

In conclusion, Cooperative Networking with link-level retransmission results in larger probability of successful reception together with increased throughput when there are small clusters, when the connectivity of the network is small ($n \leq 15, r \leq 6$), and when the probability of transmission loss is large ($p \geq 0.2$). These conditions are representative sparse sensor networks.

Our future directions include analyzing and simulating the performance and dynamic effects of link-level retransmissions on Cooperative Networking when the links or nodes fail.

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